Solutions to JEE(Main) -2021

Test Date: 20th July 2021 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. For a series LCR circuit with $R = 100\,\Omega$, $L = 0.5\,mH$ and $C = 0.1\,pF$ connected across $220V - 50Hz$ AC supply, the phase angle between current and supplied voltage and the nature of the circuit is :

(A) $\approx 90^\circ$, predominantly inductive circuit
(B) 0°, resonance circuit
(C) $\approx 90^\circ$, predominantly capacitive circuit
(D) 0°, resistive circuit

Q2. If the kinetic energy of a moving body becomes four times its initial kinetic energy, then the percentage change in its momentum will be :

(A) 100%
(B) 200%
(C) 400%
(D) 300%

Q3. If time (t), velocity (υ), and angular momentum (ℓ) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, υ, and ℓ is :

(A) $\left[t^{-2}\,\upsilon^{-1}\,\ell^{1}\right]$
(B) $\left[t^{-1}\,\upsilon^{1}\,\ell^{-2}\right]$
(C) $\left[t^{-1}\,\upsilon^{-2}\,\ell^{1}\right]$
(D) $\left[t^{1}\,\upsilon^{2}\,\ell^{-1}\right]$

Q4. An electron having de-Broglie wavelength λ is indent on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is:

(A) 0
(B) $\dfrac{2mc\,\lambda^2}{h}$
(C) $\dfrac{hc}{mc}$
(D) $\dfrac{2m^2c^2\lambda^2}{h^2}$

Q5. Two small drops of mercury each of radius R coalesce to from a single large drop. The ratio of total surface energy before and after the change is :

(A) $2^{\frac{1}{3}} : 1$
(B) $1 : 2^{\frac{1}{3}}$
(C) $2 : 1$
(D) $1 : 2$

Q6. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P} + \vec{Q}$ is n times the magnitude of $\vec{P} - \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is :

(A) $\sin^{-1}\left(\dfrac{n^2 - 1}{n^2 + 1}\right)$
(B) $\cos^{-1}\left(\dfrac{n^2 - 1}{n^2 + 1}\right)$
(C) $\sin^{-1}\left(\dfrac{n - 1}{n + 1}\right)$
(D) $\cos^{-1}\left(\dfrac{n - 1}{n + 1}\right)$

Q7. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be :

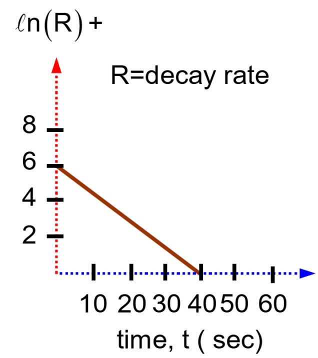
(A) $\frac{t_1 t_2}{t_2 + t_1}$
(B) $t_2 - t_1$
(C) $\frac{t_1 + t_2}{2}$
(D) $\frac{t_1 t_2}{t_2 - t_1}$

Q8. The correct relation between the degrees of freedom f and the ratio of specific heat γ is :

(A) $f = \frac{2}{\gamma + 1}$
(B) $f = \frac{2}{\gamma - 1}$
(C) $f = \frac{1}{\gamma + 1}$
(D) $f = \frac{\gamma + 1}{2}$

Q9. For a certain radioactive process the graph between $\ell n(R)$ and t(sec) is obtained as shown in the figure. Then the value of half life for the unknown radioactive material is approximately :

(A) 2.62 sec
(B) 9.15 sec
(C) 4.62 sec
(D) 6.93 sec



Q10. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

(A) Hollow cylinder
(B) Ring
(C) Solid sphere
(D) Solid cylinder

Q11. In an electromagnetic wave the electric field vector and magnetic field vector are given as $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{k}$ respectively. The direction of propagation of electromagnetic wave is along.

(A) $(-\hat{k})$
(B) $(\hat{k})$
(C) $(-\hat{j})$
(D) $\hat{j}$

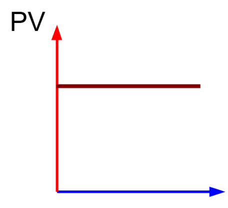
Q12. The length of a metal wire is ℓ_1, when the tension in it is T_1 and is ℓ_2 when the tension is T_2. The natural length of the wire is :

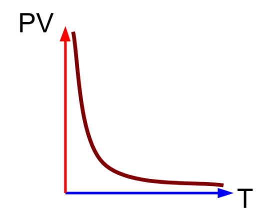
(A) $\sqrt{\ell_1 \ell_2}$
(B) $\frac{\ell_1 + \ell_2}{2}$
(C) $\frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$
(D) $\frac{\ell_1 T_2 + \ell_2 T_1}{T_2 + T_1}$

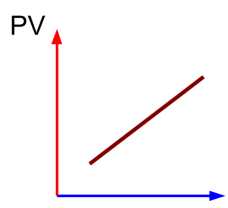
Q13. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to :

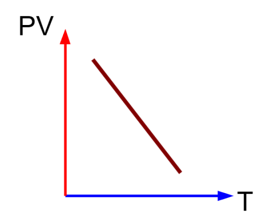
(A) $t^{\frac{3}{4}}$
(B) $t^{\frac{3}{2}}$
(C) $t^{\frac{1}{2}}$
(D) $t^{\frac{1}{4}}$

Q14. Which of the following graphs represent the behaviour of an ideal gas? Symbols have their usual meaning.

(A) 

(B) 

(C) 

(D) 

Q15. With what speed should a galaxy move outward with respect to earth so that the sodium –D line at wavelength 5890 $\mathring{A}$ is observed at 5896 $\mathring{A}$?

(A) 322 km / sec
(B) 296 km / sec
(C) 306 km / sec
(D) 336 km / sec

Q16. A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are υ_1 and υ_2 respectively. The time period of its oscillation is given as :

(A) $T = 2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{\upsilon_1^2 - \upsilon_2^2}}$
(B) $T = 2\pi\sqrt{\dfrac{x_2^2 + x_1^2}{\upsilon_1^2 - \upsilon_2^2}}$
(C) $T = 2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{\upsilon_1^2 + \upsilon_2^2}}$
(D) $T = 2\pi\sqrt{\dfrac{x_2^2 + x_1^2}{\upsilon_1^2 + \upsilon_2^2}}$

Q17. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellites is :

(A) 0.7
(B) 3.0
(C) 2.0
(D) 1.5

Q18. Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then :

(A) $T_A = T_B$
(B) $T_A > T_B \left(\text{if } m_A > m_B\right)$
(C) $\dfrac{T_A}{T_B} = \left(\dfrac{r_A}{r_B}\right)^{\frac{3}{2}}$
(D) $T_A > T_B \left(\text{if } r_A > r_B\right)$

Q19. At an angle of 30° to the magnetic meridian, the apparent dip is 45°. Find the true dip:

(A) $\tan^{-1}\frac{\sqrt{3}}{2}$ (B) $\tan^{-1}\sqrt{3}$

(C) $\tan^{-1}\frac{1}{\sqrt{3}}$ (D) $\tan^{-1}\frac{2}{\sqrt{3}}$

Q20. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi\times10^{-7}\,H/m$. Absolute permeability of the material of the rod is:

(A) $3\pi\times10^{-4}\,H/m$ (B) $\pi\times10^{-4}\,H/m$

(C) $2\pi\times10^{-4}\,H/m$ (D) $4\pi\times10^{-4}\,H/m$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. A radioactive substance decays to $\left(\frac{1}{16}\right)^{th}$ of this initial activity in 80 days. The half life of the radioactive substance expressed in days is ___________.

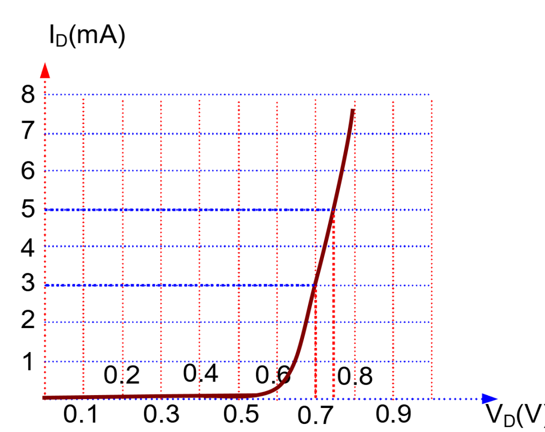
Q2. A certain metallic surface is illuminated by monochromatic radiation of wavelength λ. The stopping potential for photoelectric current for this radiation is $3V_0$. If the same surface is illuminated with a radiation of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is ________ λ.

Q3. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is __________.

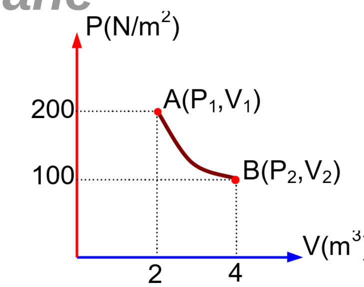
Q4. A series LCR circuit of $R = 5\Omega$, $L = 20\,mH$ and $C = 0.5\,\mu F$ is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is ________ $\times 10^2$ W.

Q5. A body rotating with an angular speed of $600\,rpm$ is uniformly accelerated to $1800\,rpm$ in 10 sec. The number of rotations made in the process is ___________.

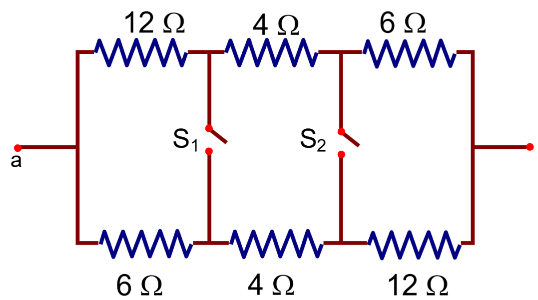
Q6. For the forward biased diode characteristics shown in the figure, the dynamic resistance at $I_D = 3\,mA$ will be __________ Ω.



Q7. One mole of an ideal gas at $27^\circ C$ is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be ________ $\times 10^{-1}$ J.
[Given : $R = 8.3\,J/mole\,K$, $\ell n2 = 0.6931$]
(Round off to the nearest integer)

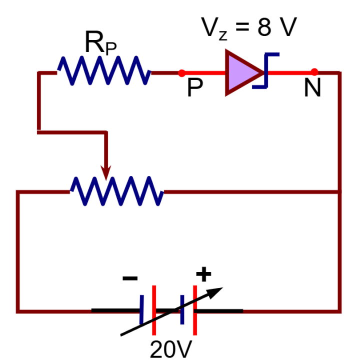


Q8. In the given figure switches S_1 and S_2 are in open condition. The resistance across ab when the switches S_1 and S_2 are closed is _________ Ω.



Q9. A body of mass 'm' is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is ___________.

Q10. A zener diode having zener voltage 8 V and power dissipation rating of $0.5\,W$ is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance R_p is __________ Ω.



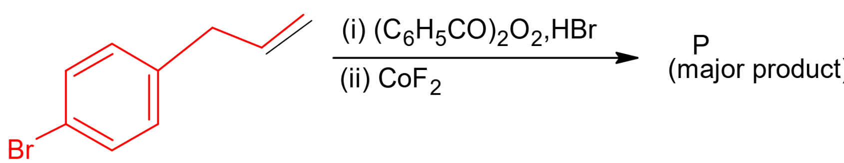
PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

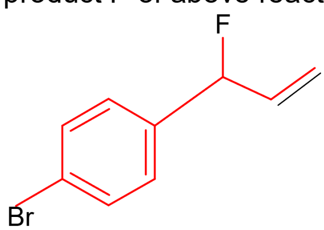
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1.

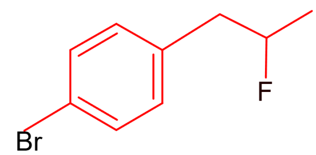


Major product P of above reaction, is :

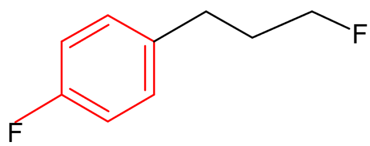
(A)



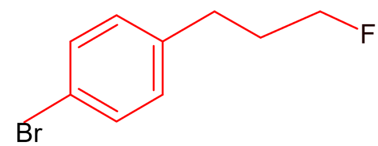
(B)



(C)



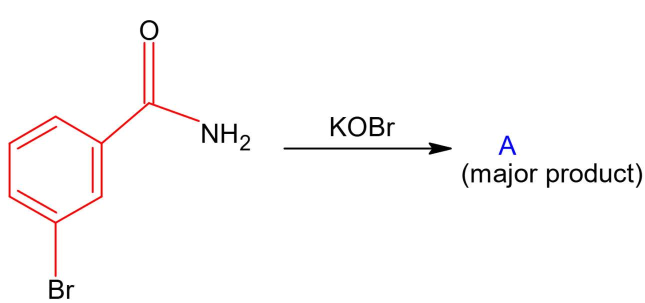
(D)

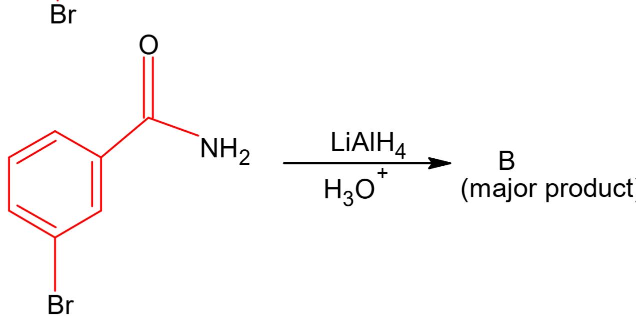


Q2. Outermost electronic configuration of a group 13 element, E, is $4s^2, 4p^1$. The electronic configuration of an element of p-block period-five placed diagonally to element, E is:

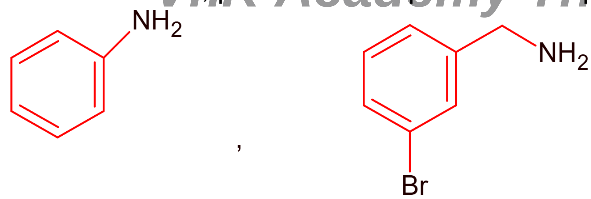
(A) $[Kr]4d^{10}5s^2 5p^2$
(B) $[Kr]3d^{10}4s^2 4p^2$
(C) $[Xe]5d^{10}6s^2 6p^2$
(D) $[Ar]3d^{10}4s^2 4p^2$

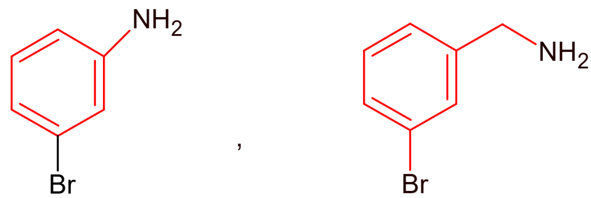
Q3.

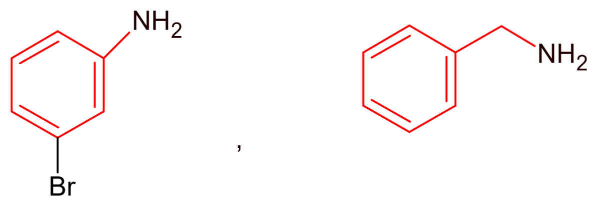


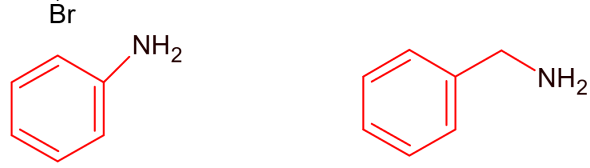


In the above reactions, product A and product B respectively are:

(A) 

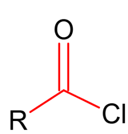
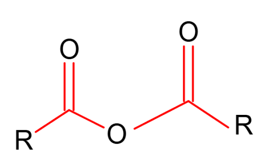
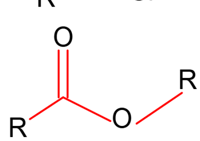
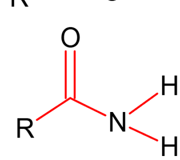
(B) 

(C) 

(D) 

Q4. Bakelite is a cross- linked polymer of formaldehyde and :
(A) PHBV
(B) Buna – S
(C) Novolac
(D) Dacron

Q5. Consider two chemical reactions (A) and (B) that take place during metallurgical process:
(a) $ZnCO_{3(s)} \xrightarrow{\Delta} ZnO_{(s)} + CO_{2(g)}$
(b) $2ZnS_{(s)} + 3O_{2(g)} \xrightarrow{\Delta} 2ZnO_{(s)} + 2SO_{2(g)}$
The correct option of names given to them respectively is :
(A) (a) is calcination and (b) is roasting
(B) Both (a) and (b) are producing same product so both are calcination
(C) Both (a) and (b) are producing same product so both are roasting
(D) (a) is roasting and (b) is calcination

Q6.
(a) 
(b) 
(c) 
(d) 

The correct order of their reactivity towards hydrolysis at room temperature is:
(A) (d) > (b) > (a) > (c)
(B) (d) > (a) > (b) > (c)
(C) (a) > (c) > (b) > (d)
(D) (a) > (b) > (c) > (d)

Q7. Spin only magnetic moment of an octahedral complex of Fe^{2+} in the presence of a strong field ligand in BM is :

(A) 2.82 (B) 3.46

(C) 0 (D) 4.89

Q8. Cu^{2+} salt reacts with potassium iodide to give :

(A) Cu_2I_3 (B) Cu_2I_2

(C) $Cu(I_3)_2$ (D) CuI

Q9. Which one of the following statements is not true about enzymes ?

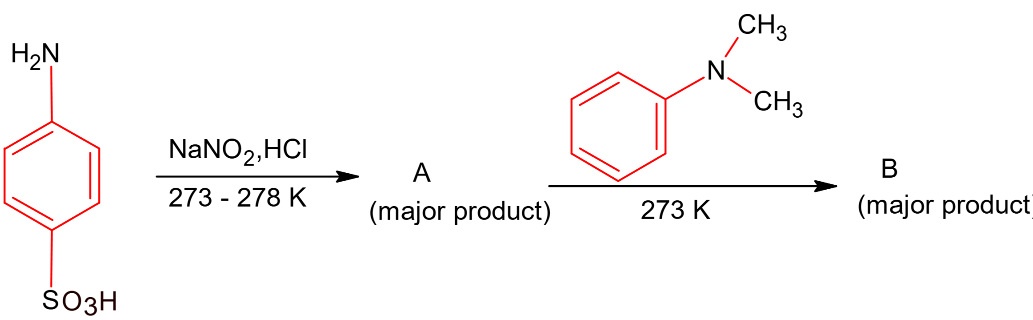
(A) The action of enzymes is temperature and pH specific.

(B) Enzymes are non-specific for a reaction and substrate.

(C) Enzymes work as catalysts by lowering the activation energy of a biochemical reaction.

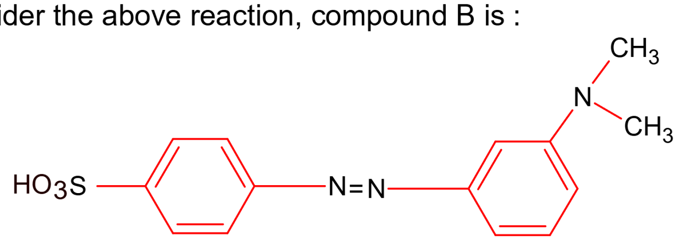
(D) Almost all enzymes are proteins.

Q10.

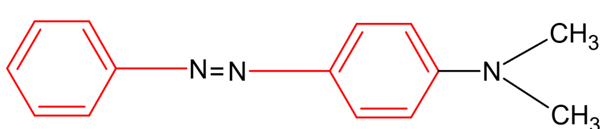


Consider the above reaction, compound B is :

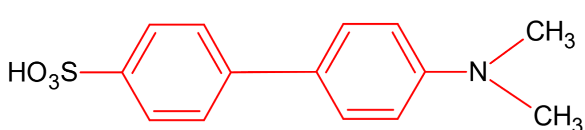
(A)



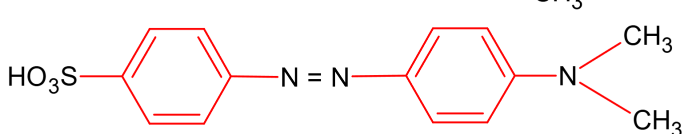
(B)



(C)



(D)



Q11. Which one of the following gases is reported to retard photosynthesis ?

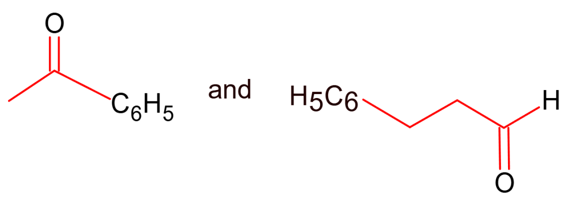
(A) CO_2 (B) CFCs

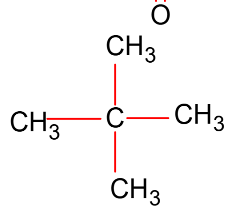
(C) CO (D) NO_2

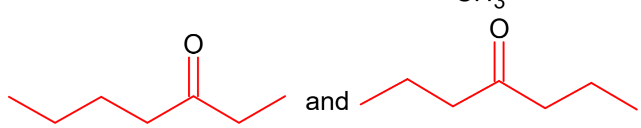
Q12. Benzene on nitration gives nitrobenzene in presence of HNO_3 and H_2SO_4 mixture, where:
(A) both H_2SO_4 and HNO_3 act as a bases
(B) both H_2SO_4 and HNO_3 act as an acids
(C) HNO_3 acts as a base and H_2SO_4 acts as an acid
(D) HNO_3 acts as an acid and H_2SO_4 acts as a base

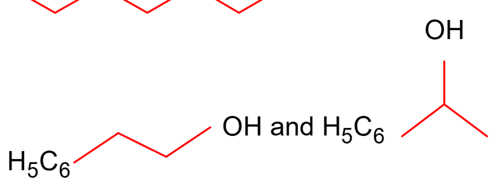
Q13. Which one of the following species doesn't have a magnetic moment of 1.73 BM, (spin only value)?
(A) CuI
(B) $[Cu(NH_3)_4]Cl_2$
(C) O_2^-
(D) O_2^+

Q14. Which one of the following pairs of isomers is an example of metamerism?

(A)


(B) $CH_3\ CH_2\ CH_2\ CH_2\ CH_3$ and


(C)


(D)


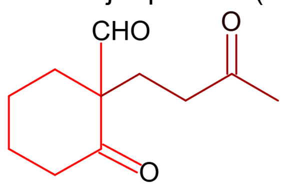
Q15. The hybridisations of the atomic orbitals of nitrogen in NO_2^-, NO_2^+ and NH_4^+ respectively are :
(A) sp^3, sp and sp^2
(B) sp, sp^2 and sp^3
(C) sp^2, sp and sp^3
(D) sp^3, sp^2 and sp

Q16. Metallic sodium does not react normally with :
(A) gaseous ammonia
(B) tert – butyl alcohol
(C) But – 2 – yne
(D) Ethyne

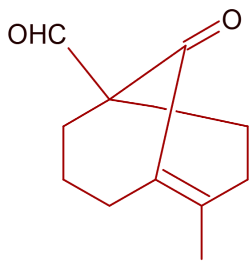
Q17. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of :
(A) $AgNO_3$
(B) HNO_3
(C) $BaSO_4$
(D) $CuSO_4$

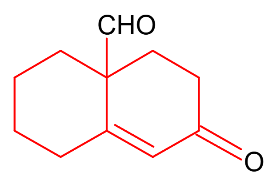
Q18. The single largest industrial application of dihydrogen is :
(A) Rocket fuel in space research
(B) In the synthesis of nitric acid
(C) In the synthesis of ammonia
(D) Manufacture of metal hydrides

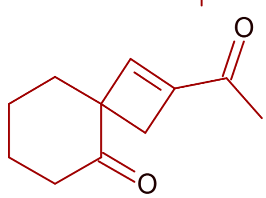
Q19. The major product (P) in the following reaction is :

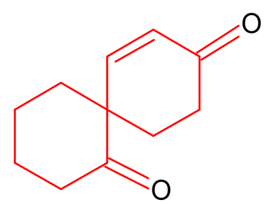


$\xrightarrow[\text{(ii) } H^+ ,\ \Delta]{\text{(i) KOH alc.}}$ P (major product)

(A)


(B)


(C)


(D)


Q20. A solution is 0.1 M in Cl^- and 0.001 M in CrO_4^{2-}. Solid $AgNO_3$ is gradually added to it. Assuming that the addition does not change in volume and $K_{sp}(AgCl) = 1.7 \times 10^{-10} M^2$ and $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12} M^3$.
Select correct statement from the following :
(A) Ag_2CrO_4 precipitates first because the amount of Ag^+ needed is low.
(B) Ag_2CrO_4 precipitates first as its K_{sp} is low.
(C) $AgCl$ will precipitate first as the amount of Ag^+ needed to precipitate is low.
(D) $AgCl$ precipitates first because its K_{sp} is high.

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. 4g equimolar mixture of $NaOH$ and Na_2CO_3 contains x g of $NaOH$ and y g of Na_2CO_3. The value of x is __________ g.(Nearest Integer)

Q2. When 0.15 g of an organic compound was analyzed using carius method for estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is ________. .(Nearest Integer) (Atomic mass: Ag = 108, Br = 80)

Q3. For a given chemical reaction $A \rightarrow B$ at 300 K the free energy change is – 49.4 kJ mol^{-1} and the enthalpy of reaction is 51.4 kJ mol^{-1}. The entropy change of the reaction is ________ J K^{-1} mol^{-1}.

Q4. The wavelength of electrons accelerated from rest through a potential difference of 40 kV is $x \times 10^{-12}$ m. The value of x is ________. .(Nearest Integer)

Given : Mass of electron $= 9.1 \times 10^{-31}$ kg

Charge on an electron $= 1.6 \times 10^{-19}$ C

Planck's constant $= 6.63 \times 10^{-34}$ Js

Q5. 100 ml of 0.0018% (w/v) solution of Cl^- ion was the minimum concentration of Cl^- required to precipitate a negative sol in one h. The coagulating value of Cl^- ion is _______. (Nearest integer)

Q6. An aqueous solution of $NiCl_2$ was heated with excess sodium cyanide in presence of strong oxidizing agent to form $\left[Ni(CN)_6\right]^{2-}$. The total change in number of unpaired electrons on metal centre is ________.

Q7. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is __________.

Q8. The vapour pressures of A and B at $25°C$ are 90mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is $x \times 10^{-1}$. The value x is _______. .(Nearest Integer)

Q9. $PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$

In the above first order reaction, the concentration of PCl_5 reduces from initial concentration $50\ mol\ L^{-1}$ to $10\ mol\ L^{-1}$ in 120 minutes at 300K. The rate constant for the reaction at 300K is $x \times 10^{-2}\ min^{-1}$. the value of x is ________.

$[\text{Given } \log 5 = 0.6989]$

Q10. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.

$6\,OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$

A current of xA has to be passed for 10 h to produce $10.0\,g$ of potassium chlorate. The value of x is ________. (Nearest Integer)

(Molar mass of $KClO_3 = 122.6\,g\,mol^{-1}, F = 96500\,C$)

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point $(2,3,-1)$ with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is one the plane P ?

(A) $(1,2,2)$ (B) $(-1,1,2)$

(C) $(1,1,1)$ (D) $(1,1,2)$

Q2. If the real part of the complex number $(1-\cos\theta+2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta\in(0,\pi)$, then the value of the integral $\int_0^{\theta}\sin x\,dx$ is equal to :

(A) -1 (B) 0

(C) 2 (D) 1

Q3. The value of $k\in R$, for which the following system of linear equations

$3x-y+4z=3$,

$x+2y-3z=-2$,

$6x+5y+kz=-3$,

has infinitely many solutions, is :

(A) 3 (B) -3

(C) -5 (D) 5

Q4. For the natural numbers m, n, if $(1-y)^m(1+y)^n=1+a_1y+a_2y^2+....+a_{m+n}\,y^{m+n}$ and $a_1=a_2=10$, then the value of $(m+n)$ is equal to :

(A) 64 (B) 88

(C) 80 (D) 100

Q5. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to :

(A) $\frac{151}{63}$ (B) $\frac{-291}{76}$

(C) $\frac{220}{21}$ (D) $\frac{-181}{69}$

Q6. If $f:R \to R$ is given by $f(x)=x+1$, then the value of

$\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+....+f\left(\frac{5(n-1)}{n}\right)\right]$, is :

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$

(C) $\frac{7}{2}$ (D) $\frac{5}{2}$

Q7. Let P be a variable point on the parabola $y=4x^2+1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line y = x is :

(A) $2(x-3y)^2+(3x-y)+2=0$ (B) $(3x-y)^2+(x-3y)+2=0$

(C) $2(3x-y)^2+(x-3y)+2=0$ (D) $(3x-y)^2+2(x-3y)+2=0$

Q8. The sum of all the local minimum values of the twice differentiable function $f:R\to R$ defined by $f(x)=x^3-3x^2-\frac{3f''(2)}{2}x+f''(1)$ is :

(A) -22 (B) 5

(C) 0 (D) -27

Q9. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4,1)$ and having their centres on the circumference of the circle $x^2+y^2+2x+4y-4=0$. If $\frac{r_1}{r_2}=a+b\sqrt{2}$, then $a+b$ is equal to :

(A) 5 (B) 11

(C) 7 (D) 3

Q10. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a-b|$ is equal to

(A) 9 (B) 7

(C) 11 (D) 1

Q11. Let $y=y(x)$ satisfies the equation $\frac{dy}{dx}-|A|=0$, for all $x>0$, where $A=\begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x}\end{bmatrix}$. If $y(\pi)=\pi+2$, then the value of $y\left(\frac{\pi}{2}\right)$ is :

(A) $\frac{\pi}{2}+\frac{4}{\pi}$ (B) $\frac{3\pi}{2}-\frac{1}{\pi}$

(C) $\frac{\pi}{2}-\frac{4}{\pi}$ (D) $\frac{\pi}{2}-\frac{1}{\pi}$

Q12. Let $f:R-\left\{\frac{\alpha}{6}\right\}\to R$ be defined by $f(x)=\frac{5x+3}{6x-\alpha}$. Then the value of α for which $(fof)(x)=x$, for all $x\in R-\left\{\frac{\alpha}{6}\right\}$, is :

(A) 6 (B) 8
(C) No such α exists (D) 5

Q13. The lines $x=ay-1=z-2$ and $x=3y-2=bz-2,\ (ab\neq 0)$ are coplanar, if :

(A) $a=2, b=2$ (B) $b=1, a\in R-\{0\}$
(C) $a=2, b=3$ (D) $a=1, b\in R-\{0\}$

Q14. In a triangle ABC, if $|\overrightarrow{BC}|=3, |\overrightarrow{CA}|=5$ and $|\overrightarrow{BA}|=7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to :

(A) $\frac{19}{2}$ (B) $\frac{13}{2}$
(C) $\frac{15}{2}$ (D) $\frac{11}{2}$

Q15. Let $g(t)=\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$, where $f(x)=\log_e\left(x+\sqrt{x^2+1}\right), x\in R$. Then which one of the following is correct?

(A) $g(1)=g(0)$ (B) $g(1)=\sqrt{2}g(0)$
(C) $g(1)+g(0)=0$ (D) $\sqrt{2}g(1)=g(0)$

Q16. Consider the following three statements :
(A) if 3 + 3 = 7 then 4 + 3 = 8.
(B) If 5 + 3 = 8 then earth is flat.
(C) If both (A) and (B) are true then 5 + 6 = 17.
Then, which of the following statements is correct?

(A) (A) and (C) are true while (B) is false (B) (A) and (B) are false while (C) is true
(C) (A) is true while (B) and (C) are false (D) (A) is false, but (B) and (C) are true

Q17. If sum of the first 21 terms of the series $\log_{9^{1/2}}x+\log_{9^{1/3}}x+\log_{9^{1/4}}x+....$, where x > 0 is 504, then x is equal to :

(A) 7 (B) 9
(C) 243 (D) 81

Q18. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2}\left[[x]-\sin x\right]dx$ is equal to :

(A) $-\pi$ (B) 0
(C) 1 (D) π

Q19. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

(A) $\frac{\sqrt{2}-1}{2}$ (B) $\frac{\sqrt{5}+1}{4}$

(C) $\frac{\sqrt{5}-1}{2}$ (D) $\frac{\sqrt{5}-1}{4}$

Q20. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is :

(A) greater than $\frac{1}{2}$ (B) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(C) exactly equal to $\frac{1}{2}$ (D) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to……..

Q2. If $\lim_{x\to 0}\frac{axe^x - \beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2 x} = 10, \alpha, \beta, \gamma \in R,$ then the value of $\alpha+\beta+\gamma$ is…….

Q3. Consider a triangle having vertices $A(-2, 3), B(1, 9)$ and $C(3, 8)$. If a line L passing through the circum-center of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is…………

Q4. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha+\beta)$ is equal to………

Q5. Let $A = \{a_{ij}\}$ be a 3 x 3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if} \quad i < j, \\ 2 & \text{if} \quad i = j, \\ (-1)^{i+j} & \text{if} \quad i > j, \end{cases}$$

then $\det\left(3\,\text{Adj}\left(2A^{-1}\right)\right)$ is equal to…….

Q6. For $p > 0$, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}\,p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta = \frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to……….

Q7. Let a curve $y = y(x)$ be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right)dx = \sqrt{e^{2x}-1}\,dy$$

If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to……..

Q8. The number of solutions of the equation $\log_{(x+1)}\left(2x^2+7x+5\right)+\log_{(2x+5)}(x+1)^2-4=0,\ x>0,$ is.........

Q9. For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)}=\sum_{k=0}^{20}\frac{A_k}{a+k}$, where $\alpha>0$. Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to...........

Q10. Let a function $g:[0,4]\to R$ be defined as $g(x)=\begin{cases}\max\limits_{0\le t\le x}\left\{t^3-6t^2+9t-3\right\}, & 0\le x\le 3\\ 4-x & ,3<x\le 4\end{cases}$, then the number of points in the interval $(0,4)$ where $g(x)$ is NOT differentiable, is.......

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**C**	2.	**A**	3.	**C**	4.	**B**
5.	**A**	6.	**B**	7.	**A**	8.	**B**
9.	**C**	10.	**D**	11.	**C**	12.	**C**
13.	**B**	14.	**C**	15.	**C**	16.	**A**
17.	**B**	18.	**A**	19.	**A**	20.	**C**

SECTION - B

1.	**20**	2.	**4**	3.	**3**	4.	**125**
5.	**200**	6.	**25**	7.	**17258**	8.	**10**
9.	**3**	10.	**192**				

PART – B (CHEMISTRY)

SECTION - A

1.	**D**	2.	**A**	3.	**B**	4.	**C**
5.	**A**	6.	**D**	7.	**C**	8.	**B**
9.	**B**	10.	**D**	11.	**D**	12.	**C**
13.	**A**	14.	**C**	15.	**C**	16.	**C**
17.	**A**	18.	**C**	19.	**D**	20.	**C**

SECTION - B

1.	**1**	2.	**68**	3.	**336**	4.	**6**
5.	**1**	6.	**2**	7.	**8**	8.	**1**
9.	**1**	10.	**1**				

PART – C (MATHEMATICS)

SECTION - A

1.	**A**	2.	**D**	3.	**C**	4.	**C**
5.	**C**	6.	**C**	7.	**C**	8.	**D**
9.	**A**	10.	**D**	11.	**A**	12.	**D**
13.	**B**	14.	**D**	15.	**D**	16.	**A**
17.	**D**	18.	**A**	19.	**C**	20.	**A**

SECTION – B

1.	**7**	2.	**3**	3.	**9**	4.	**9**
5.	**108**	6.	**6**	7.	**2**	8.	**8**
9.	**9**	10.	**1**				

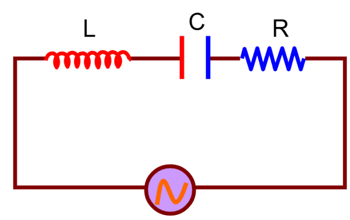
Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. $\omega = 100\pi$, $R = 100\Omega$, $X_L = L\omega = 0.5\times10^{-3}\times100\pi = 5\pi\times10^{-2}\Omega$, and

$X_C = \dfrac{1}{C\omega} = \dfrac{1}{0.1\times10^{-12}\times100\pi} = \dfrac{10^{11}}{\pi}\Omega$



Since X_C is approximately infinite , so the phase angle between current and supplied voltage and the nature of the circuit is $\approx 90^\circ$, predominantly capacitive circuit

Sol2. $p = \sqrt{2mK} \Rightarrow \dfrac{p_2}{p_1} = \sqrt{\dfrac{K_2}{K_1}} \Rightarrow \dfrac{p_2}{p} = \sqrt{\dfrac{4K_1}{K_1}} = 2 \Rightarrow p_2 = 2p$

The percentage change in its momentum $= \dfrac{\Delta p}{p}\times100 = \dfrac{p}{p}\times100 = 100\%$

Sol3. Let Mass $\alpha \left[t^a \upsilon^b \ell^c\right] \Rightarrow \left[M^1L^0T^0\right] \equiv T^a \times \left[LT^{-1}\right]^b \times \left[ML^2T^{-1}\right]^c$

$\Rightarrow a - b - c = 0 \quad c = 1$, and $b + 2c = 0 \Rightarrow b = -2c = -2$

$\Rightarrow a = b + c = 1 - 2 = -1$

Sol4. According to de-Broglie 's hypothesis

$\lambda = \dfrac{h}{mv} \Rightarrow p = mv = \dfrac{h}{\lambda} \Rightarrow K = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$

Cut-off wavelength of emitted X-ray $= \dfrac{hc}{K} = hc\times\dfrac{2m\lambda^2}{h^2} = \dfrac{2mc\lambda^2}{h}$ **[IIT Adv]**

Sol5. Total surface energy before coalesce $= U_i = 2\times4\pi R^2\times\sigma = 8\pi R^2\sigma$ ....(i)

Let new radius becomes r ,so according to conservation energy we can write

$2\times\dfrac{4\pi R^3}{3} = \dfrac{4\pi r^3}{3} \Rightarrow r = 2^{\frac{1}{3}}R$

Total surface energy after coalesce $= U_f = 4\pi r^2\times\sigma = 2^{\frac{2}{3}}\times4\pi R^2\sigma$ ....(ii)

$\Rightarrow \dfrac{U_i}{U_f} = \dfrac{8\pi R^2\sigma}{2^{\frac{2}{3}}\times4\pi R^2\sigma} = 2^{\frac{1}{3}} : 1$

Sol6. $\left|\vec{P}\right| = \left|\vec{Q}\right| = x$

let the angle between $\vec{P}$ and $\vec{Q}$ is θ, so according to question , we can write

$$\left|\vec{P}+\vec{Q}\right| = n\times\left|\vec{P}-\vec{Q}\right| \Rightarrow \sqrt{P^2+Q^2+2PQ\cos\theta} = n\times\sqrt{P^2+Q^2-2PQ\cos\theta}$$

$$\Rightarrow \sqrt{x^2+x^2+2x^2\cos\theta} = n\times\sqrt{x^2+x^2-2x^2\cos\theta}$$

$$\Rightarrow 2+2\cos\theta = 2n^2-2n^2\cos\theta \Rightarrow \left(1+n^2\right)\cos\theta = n^2-1 \Rightarrow \cos\theta = \frac{n^2-1}{n^2+1}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

Sol7. let the distance travelled is x, so

$$\left|\vec{v}_{Eg}\right| = \frac{x}{t_2} \Rightarrow \text{speed of escalator with respect to ground}$$

$$\left|\vec{v}_{BE}\right| = \frac{x}{t_1} \Rightarrow \text{speed of boy with respect to escalator}$$

$$\left|\vec{v}_{Bg}\right| = \frac{x}{t_1}+\frac{x}{t_2} \Rightarrow \text{ speed of boy with respect to ground}$$

The time taken by him to walk up on the moving escalator $= t = \frac{x}{\left|\vec{v}_{Bg}\right|}$

$$\Rightarrow t = \frac{x}{\frac{x}{t_1}+\frac{x}{t_2}} = \frac{t_1t_2}{t_1+t_2}$$

Sol8. $\gamma = 1+\frac{2}{f} \Rightarrow \frac{2}{f} = \gamma-1 \Rightarrow f = \frac{2}{\gamma-1}$

Sol9. $R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} \Rightarrow \ell n(R) = \ell n\left(\lambda N_0\right)-\lambda t$, so

$$\lambda = \tan\theta = \frac{6}{40} = \frac{3}{20}s^{-1} \Rightarrow \text{Slope of graph}$$

$$\Rightarrow t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693\times 20}{3} = 4.62\,s$$

Sol10. $K_T = \frac{mv^2}{2}$ and $K_R = \frac{I\omega^2}{2} = \frac{Iv^2}{2R^2}$

According to question, we can write

$$K_R = \frac{1}{2}K_T \Rightarrow \frac{Iv^2}{2R^2} = \frac{1}{2}\times\frac{mv^2}{2} \Rightarrow I = \frac{mR^2}{2} \Rightarrow \text{ Solid cylinder}$$

Sol11. **As we know that direction of propagation $(\hat{p})$ of wave is given as**

$$\hat{p} = \hat{E}\times\hat{B} = \hat{i}\times\hat{k} = \left(-\hat{j}\right)$$

Sol12. Assuming Tension in metal wire suspended from roof varies linearly and 0 and T_0 (developed due its own weight) are tensions are ends of wire of length ℓ. So

$T_1 \;\alpha\; \left(\ell_1-\ell\right)$, and $T_2 \;\alpha\; \left(\ell_2-\ell\right)$

$$\frac{T_1}{T_2}=\frac{\ell_1-\ell}{\ell_2-\ell} \Rightarrow \ell=\frac{T_2\ell_1-T_1\ell_2}{T_2-T_1}$$

Sol13. **According to question, we can write**

$$\text{Power}=Fv=c \Rightarrow mv\frac{dv}{dt}=c \Rightarrow \int_0^v vdv=\frac{c}{m}\int_0^t dt \Rightarrow \frac{v^2}{2}=\frac{c}{m}t \Rightarrow v=\frac{dx}{dt}=\sqrt{\frac{2c}{m}}\,t^{\frac{1}{2}}$$

$$\Rightarrow \int_0^x dx=\sqrt{\frac{2c}{m}}\int_0^d t^{\frac{1}{2}}\,dt \Rightarrow x=\sqrt{\frac{2c}{m}}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)=\frac{2}{3}\sqrt{\frac{2c}{m}}\left(t^{\frac{3}{2}}\right) \Rightarrow x\ \alpha\left(t^{\frac{3}{2}}\right)$$

Sol14. According to ideal gas law we know that

$PV=nRT$

Sol15. **According to Doppler's effect in light , we can write**

$$\frac{v}{c}=\frac{\Delta\lambda}{\lambda} \Rightarrow \frac{v}{c}=\frac{5896-5890}{5890}=\frac{6}{5890}=\frac{3}{2945} \Rightarrow v=\frac{3}{2945}\times3\times10^5 \approx 305.6\ km/s$$

Sol16. As we know that $\upsilon^2=\omega^2\left(A^2-x^2\right)$ for SHM , so

$\upsilon_1^2=\omega^2\left(A^2-x_1^2\right)$.........(i) , and $\upsilon_2^2=\omega^2\left(A^2-x_2^2\right)$.........(ii)

Subtracting equation (ii) from equation (i) , we have

$$\upsilon_1^2-\upsilon_2^2=\omega^2\left(x_2^2-x_1^2\right) \Rightarrow \omega=\frac{2\pi}{T}=\sqrt{\frac{\upsilon_1^2-\upsilon_2^2}{x_2^2-x_1^2}} \Rightarrow T=2\pi\sqrt{\frac{x_2^2-x_1^2}{\upsilon_1^2-\upsilon_2^2}}$$

Sol17. Time period of Satellite $=T=\frac{2\pi R}{v}=\left(\frac{4\pi^2R^3}{GM}\right)^{\frac{1}{2}}$

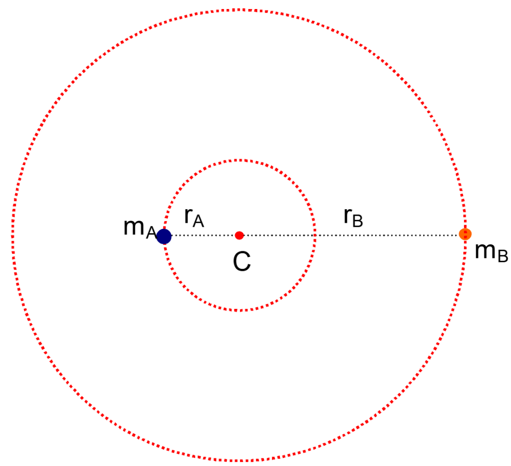
$$\Rightarrow T\ \alpha\ R^{\frac{3}{2}} \Rightarrow \frac{T_2}{T_1}=\frac{(1.02R)^{\frac{3}{2}}}{R^{\frac{3}{2}}}=(1.02)^{\frac{3}{2}}=(1+0.02)^{\frac{3}{2}}$$

$$\Rightarrow \frac{T_2}{T_1}=\left(1+\frac{3}{2}\times0.02\right) \Rightarrow T_2=(1+0.03)T_1 \Rightarrow \Delta T=(0.03)T_1$$

The percentage difference in the time periods of the two satellites$=\frac{\Delta T}{T_1}\times100$

$=(0.03)\times100=3\%$

Sol18. Since binary mass system performs circular motion about is common centre of mass, so



$$m_A\omega_A^2 r_A = \frac{Gm_Bm_A}{(r_A+r_B)^2} = \frac{Gm_Bm_A}{r^2}$$

$$\Rightarrow m_A\omega_A^2 \times \frac{m_B}{(m_A+m_B)} r = \frac{Gm_Bm_A}{r^2}$$

$$\Rightarrow \omega_A^2 = \frac{G(m_A+m_B)}{r^3}$$

$$\Rightarrow \omega_A = \sqrt{\frac{G(m_A+m_B)}{r^3}}$$

Similarly we can show that

$$\omega_B = \sqrt{\frac{G(m_A+m_B)}{r^3}}$$

Hence their angular velocity will be same , time period will be same, i.e. $T_A = T_B$

Sol19. Let the true dip is δ and apparent dip is δ' , so

$$\tan\delta' = \frac{B_V}{B_H\cos\theta} = \frac{\tan\delta}{\cos\theta} \Rightarrow \tan\delta = \tan\delta'\cos\theta = \tan 45^0 \cos 30^0 = 1\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \delta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Sol20. Given $\chi = 499$ and $\mu_0 = 4\pi\times10^{-7}\ H/m$

As we know that

$\mu_r = 1+\chi = 1+499 = 500 \Rightarrow$ Relative permeability

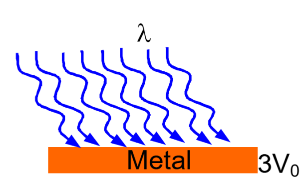
Absolute permeability= $\mu = \mu_r\mu_0 = 500\times4\pi\times10^{-7} = 2\pi\times10^{-4}\ H/m$

SECTION - B

Sol1. $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} \Rightarrow \lambda\frac{N_0}{16} = \lambda N_0 e^{-80\lambda} \Rightarrow \frac{1}{16} = e^{-80\lambda} \Rightarrow \frac{1}{16} = e^{-80\lambda}$

$$\Rightarrow 80\lambda = \ell n(16) = 4\ell n(2) \Rightarrow t_{\frac{1}{2}} = \frac{\ell n(2)}{\lambda} = \frac{80}{4} = 20\,\text{days}$$

Sol2. Let the work function of metal is ϕ, so



$$K_1 = 3V_0e = \frac{hc}{\lambda} - \phi \text{ ....(i)}$$

$$K_2 = V_0e = \frac{hc}{2\lambda} - \phi \text{ ....(ii)}$$

Doing (i) – 3 X (ii) ,we have

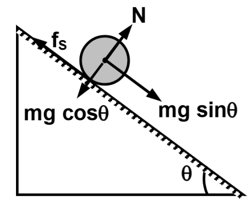
$$0 = \frac{hc}{\lambda} - \frac{3hc}{2\lambda} + 2\phi \Rightarrow \phi = \frac{hc}{4\lambda}$$

The threshold wavelength= $= \frac{hc}{\phi} = 4\lambda$

Sol3. Consider translational motion of a body

$$mg\sin\theta - f_S = ma \text{ ....(i)}$$

Consider rotational motion of body about its centre of mass



FBD of Body

$$f_S R = I\alpha \Rightarrow \alpha = \frac{f_S R}{I} \text{ ....(ii)}$$

Using the condition of rolling without slipping at contact, we have

$$\alpha R = a \Rightarrow \frac{f_S R^2}{I} = \frac{mg\sin\theta - f_S}{m} \Rightarrow f_S = \frac{Img\sin\theta}{I + mR^2}$$

Using the condition of static friction, we have

$$f_S \le \mu N \Rightarrow \frac{Img\sin\theta}{I + mR^2} \le \mu mg\cos\theta \Rightarrow \mu \ge \frac{I\tan\theta}{I + mR^2} \text{ ....(iii)}$$

$$\alpha = \frac{mgR\sin\theta}{I + mR^2} \Rightarrow a = \alpha R = \frac{mgR^2\sin\theta}{I + mR^2} = \frac{gR^2\sin\theta}{k^2 + R^2} \text{ ....(iv)}$$

$$t = \sqrt{\frac{2\ell}{a}} \Rightarrow \text{Time required to reach the ground}$$

$$v = at = \sqrt{2a\ell} = \sqrt{\frac{2g\ell R^2\sin\theta}{k^2 + R^2}} \Rightarrow \text{Speed at bottom}$$

$$\Rightarrow \frac{v_{ring}}{v_{cylinder}} = \sqrt{\frac{2g\ell R^2\sin\theta}{k_{ring}^2 + R^2}} \times \sqrt{\frac{k_{cylinder}^2 + R^2}{2g\ell R^2\sin\theta}} = \sqrt{\frac{\frac{R^2}{2} + R^2}{R^2 + R^2}} = \frac{\sqrt{3}}{2}$$

Sol4. Power dissipated in cycle $= P = \frac{V_0 I_0}{2}\cos\phi = \frac{V_0^2}{2z} \times \frac{R}{z} = \frac{V_0^2 R}{2z^2}$

For resonance, z = R, so

$$\Rightarrow P = \frac{V_0^2}{2R} = \frac{250\sqrt{2} \times 250\sqrt{2}}{2 \times 5} = 12500 \text{ Watt} = 125 \times 10^2 \text{ Watt}$$

Sol5. $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \left(\frac{2\pi}{60} \times 1800\right)^2 = \left(\frac{2\pi}{60} \times 600\right)^2 + 2\left(\frac{2\pi}{60} \times \frac{1200}{10}\right)\theta$

$$\Rightarrow (60\pi)^2 - (20\pi)^2 = 2(4\pi)\theta \Rightarrow \theta = \frac{80\pi \times 40\pi}{2 \times 4\pi} = 400\pi \text{ rad}$$

Number of rotations $= \frac{\theta}{2\pi} = \frac{400\pi}{2\pi} = 200$

Sol6. $R = \frac{\Delta V}{\Delta I} = \frac{0.75 - 0.70}{(5-3) \times 10^{-3}} = \frac{0.05 \times 1000}{2} = 25\Omega$

Sol7. Since process is isothermal, so

$$W = nRT\ell n\left(\frac{V_f}{V_i}\right) = = 1 \times 8.3 \times 300 \times \ell n\left(\frac{4}{2}\right) = 1 \times 8.3 \times 300 \times \ell n(2)$$

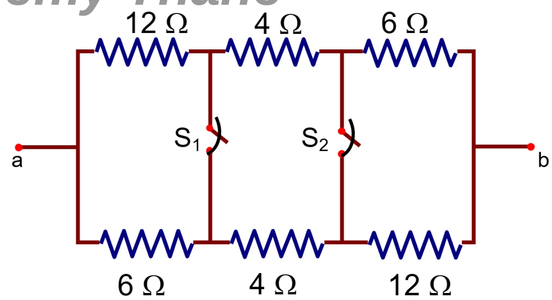
$$\Rightarrow W = 1 \times 8.3 \times 300 \times 0.6931 \approx 1725.82\,J \approx 17258 \times 10^{-1}\,J$$

Sol8. Here

$$R_{eq} = \left(12 /\!/^{\ell}\, 6\right) + \left(4 /\!/^{\ell}\, 4\right) + \left(6 /\!/^{\ell}\, 12\right)$$

$$= \left(\frac{12 \times 6}{18}\right) + \left(\frac{4 \times 4}{8}\right) + \left(\frac{6 \times 12}{18}\right)$$

$$= 4 + 2 + 4 = 10\Omega$$



Sol9. **For ascent**

$$t_a = \sqrt{\frac{2\ell}{a_{ascent}}} = \sqrt{\frac{2\ell}{g(\sin\theta + \mu\cos\theta)}}$$

For descent

$$t_d = \sqrt{\frac{2\ell}{a_{descent}}} = \sqrt{\frac{2\ell}{g(\sin\theta - \mu\cos\theta)}}$$

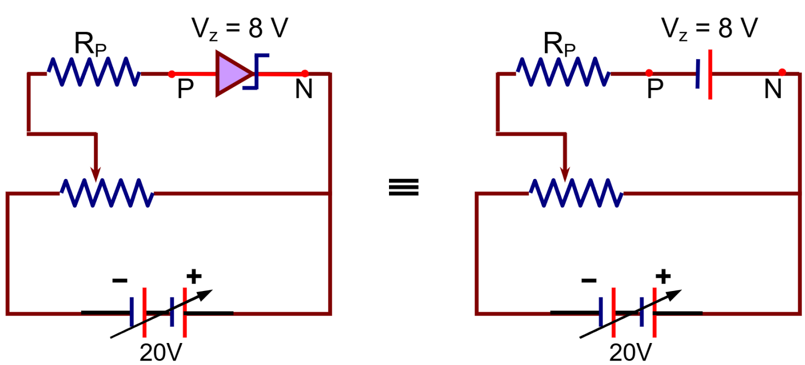
According question , we can write

$$t_a = \frac{1}{2} t_d \Rightarrow \frac{1}{\sin\theta + \mu\cos\theta} = \left(\frac{1}{4}\right)\left(\frac{1}{\sin\theta - \mu\cos\theta}\right)$$

$$\Rightarrow \sin\theta + \mu\cos\theta = 4\sin\theta - 4\mu\cos\theta \Rightarrow 5\mu\cos\theta = 3\sin\theta$$

$$\Rightarrow \mu = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 30^0 = \frac{\sqrt{3}}{5}$$

Sol10.



$$P_Z = V_Z I_Z \Rightarrow 0.5 = 8 I_Z \Rightarrow I_Z = I_P = \frac{1}{16} \text{ Amp}$$

When zener is connected across a potential divider arranged with maximum potential drop across zener diode, then

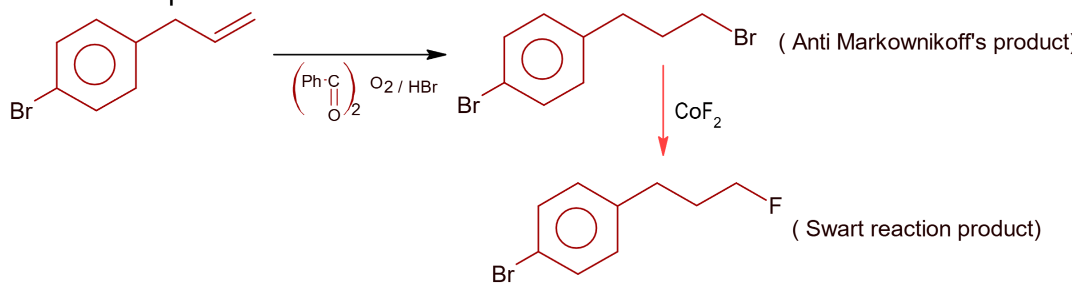
$V_P = V - V_Z = 20 - 8 = 12$ volt $\Rightarrow$ Potential difference across protective resistance R_p

$$\Rightarrow R_P = \frac{V_P}{I_P} = \frac{12}{\frac{1}{6}} = 192 \text{ Amp.}$$

PART – B (CHEMISTRY)

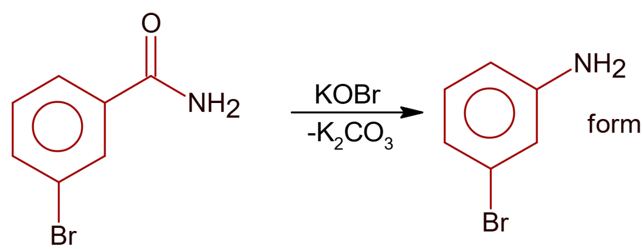
SECTION - A

Sol1. Mechanism path is free radical.

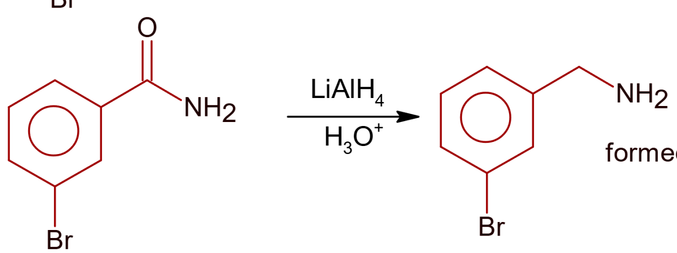


Sol2. Given electronic configuration is for Ga and in 5^{th} period diagonally situated element is Sn with respect to Ga. Hence electronic configuration of Sn is [Kr] $4d^{10}\ 5s^2\ 5p^2$.

Sol3.



formed by Hofmann's Bromamide degradation



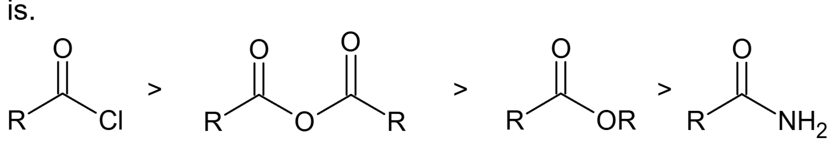
formed by hydride reagent reduction reaction

Sol4. Bakelite is cross linked polymer of formaldehyde and novolac. Novolac is linear polymer of phenol- formaldehyde.

Sol5. Calcinations is the decomposition in absence of air, $ZnCO_3 \xrightarrow[\Delta]{} ZnO + CO_2 \uparrow$

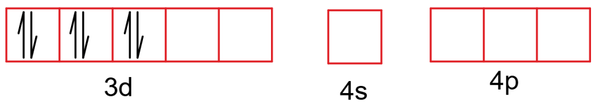
while roasting is the oxidation process, $2ZnS + 3O_2 \longrightarrow 2ZnO + 2SO_2 \uparrow$

Sol6. High leaving tendency corresponds to high reactivity towards hydrolysis. Hence order is.



Sol7. In strong field of octahedral complex of Fe^{2+}, the electronic configuration is , $Fe^{2+}_{26} \rightarrow [Ar]3d^6 4s^0$

Number of unpaired electrons are zero in the presence of strong field ligand.



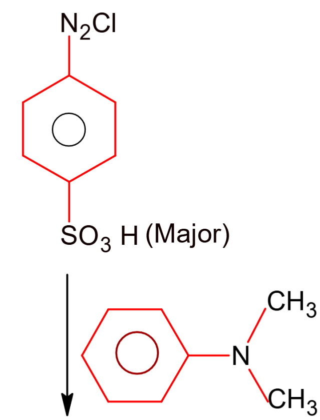
Hence, spin only magnetic moment $= \sqrt{n(n+2)} = 0BM\ (n=0)$

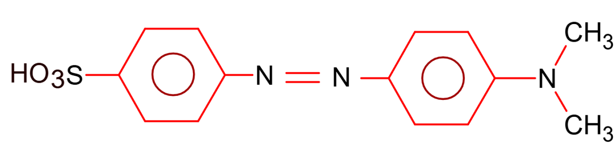
Sol8. KI acts as a reducing agent for $Cu^{2+}[2Cu^{++} + 4KI \longrightarrow Cu_2I_2 + I_2 + 4K^+]$

Sol9. Enzymes are the form of proteins and highly specific regarding temperature and pH change. It also lowers the activation energy for biochemical reaction.

Sol10.

After diazotization reaction, A is =

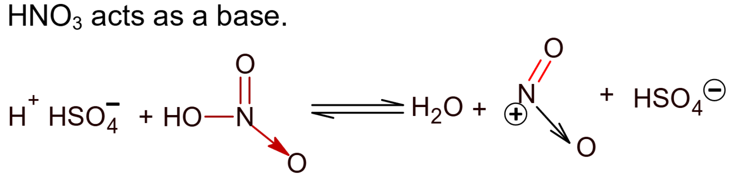




Product formed after EAS reaction (Coupling reaction)

Sol11. NO_2 gas reported to retard photosynthesis

Sol12. Due to the larger acid dissociation constant (Ka) sulphuric acid acts as an acid and HNO_3 acts as a base.



Sol13. In CuI, Cu^+ has $3d^{10} 4s^0$, $[n = 0 \text{ and } \mu = 0BM]$

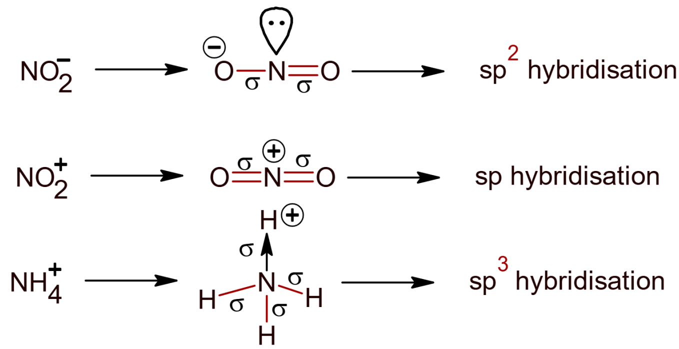
In $[Cu(NH_3)_4]Cl_2, Cu^{++}$ has $3d^9 4s^0 [n = 1 \text{ and } \mu = 1.73BM]$

In O_2^- & O_2^+ only one electron is unpaired in anti bonding MO,

Hence $[n = 1 \text{ and } \mu = 1.73\text{ BM}]$

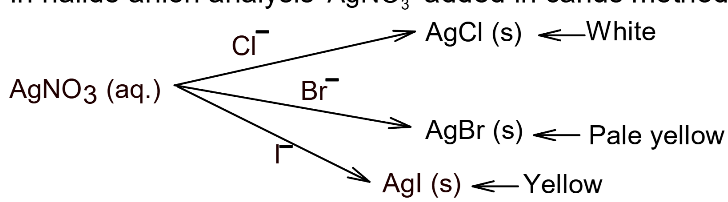
Sol14. In metamers, distribution of alkyl groups are changed with respect to polyvalent functional groups.

Sol15.



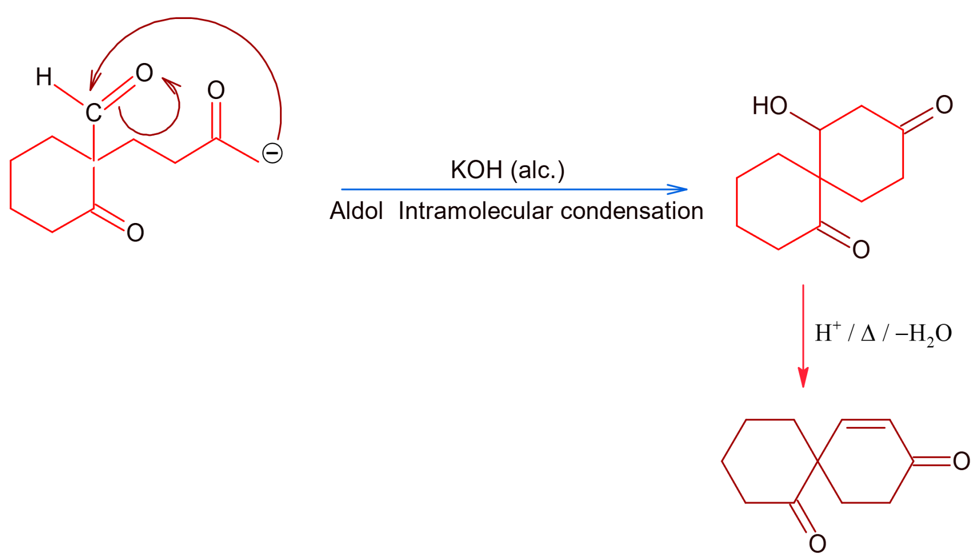
Sol16. Due to the absence of acidic hydrogen in But – 2- yne, metallic sodium does not react with this .

Sol17. In halide anion analysis $AgNO_3$ added in carius method to detect halide anion.



Sol18. Regarding industrial consumption of H_2 manufacturing of NH_3 is the largest applications.

Sol19.



Sol20. $\left[Cl^{-}\right] = 10^{-1}\ M$

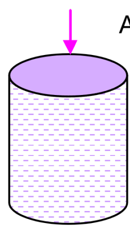
$\left[CrO_4^{--}\right] = 10^{-3}\ M$

for AgCl ppt[1], $\left[Ag^{+}\right]_{req} = \dfrac{1.7 \times 10^{-10}}{10^{-1}} M$

$\left[Ag^{+}\right]_{req} = 1.7 \times 10^{-9}\ M$

For Ag_2CrO_4 ppt[2], $\left[Ag^{+}\right]_{req} = \sqrt{\dfrac{1.9 \times 10^{-12}}{10^{-3}}} M$

$\left[Ag^{+}\right]_{req} = 4.3 \times 10^{-5} M$



$AgNO_3$ (Solid) Solution of

Solution of Cl^{-} & CrO_4^{2-}

Being lower concentration of $[Ag^{+}]$ in case of AgCl, it will precipitate first.

SECTION - B

Sol1. Total wt $= 4\,gm = W_{NaOH} + W_{Na_2CO_3}$

Let us suppose moles are ' m ' for each.

4 = 40m + 106 m

$\therefore\ m = \left(\dfrac{4}{146}\right)$ mole of NaOH and Na_2CO_3 each.

$\therefore$ Mass of NaOH (x gm) $= \dfrac{4}{146} \times 40 = 1.095 \approx 1.0$

Sol2. 0.15 gm (organic compounds) $\xrightarrow{Ag\,NO_3}$ AgBr (s)

$\downarrow$

0.2397 gm

Weight of Br in AgBr $= \left(\dfrac{80}{188} \times 0.2397\right) = 0.102\,gm$

% Br in compound $= \dfrac{0.102 \times 100}{0.15} = 68\%$

Sol3. From $\Delta H = \Delta G + T\Delta S\ \therefore\ \Delta S = \dfrac{\Delta H - \Delta G}{T}$

$\therefore\ \Delta S = \left[\dfrac{51.4 - (-49.4)}{300}\right] \times 1000\ JK^{-1}mol^{-1} = 336\,JK^{-1}mol^{-1}$

Sol4. Here, $\lambda = \dfrac{h}{\sqrt{2 \times m \times eV}} = \dfrac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 40000 \times 9.1 \times 10^{-31}}}$ m

$\therefore\ \lambda = 0.614 \times 10^{-11} m$

$\lambda = 6.14 \times 10^{-12} m$ [Nearest integer = 6]

Sol5. Wt of Cl^- in 100 ml = 1.8×10^{-3} gm

Mol . of Cl^- in 100 ml $= \dfrac{1.8\times10^{-3}}{35.5} = 0.0507\times10^{-3}$ mole

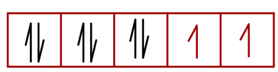
$\therefore \left[Cl^-\right] = \dfrac{0.0507\times10^{-3}\times1000}{100} = 5.07\times10^{-4}M$

i.e. 0.507 milli mole in one lit required in one hr.
Coagulation value = (milli mole / lit) required in one hr $= 0.507$
= 1 (Nearest integer)

Sol6. $NiCl_2 + NaCN \xrightarrow[\text{O.A.}]{\text{strong}} \left[Ni(CN)_6\right]^{2-}$

Complex has Ni^{4+} and strong ligand, hence following are the metal ion electronic configuration.

$Ni^{2+} \longrightarrow 3d^8\ 4s^0$



Ni^{4+} (strong ligand)

$3d^6\quad 4s^0$

Change of unpaired electron = 2

Sol7. In fcc structure of diamond four C present in fcc lattice and other four C present in tetrahedral voids where 50% of tetrahedral voids are occupied. Hence number of carbon atoms present per unit cell of diamond is 8.

Sol8. Total pressure of mixture $= P_{Total} = X_A\times P_A^o + X_B\times P_B^o = 90\times0.6+15\times0.4 = 60mm$

Mole fraction of B in vapour phase , $(X'_B) = \dfrac{X_B\times P_B^o}{P_{Total}}$

$\therefore X'_B = \dfrac{0.4\times15}{60} = 0.1 = 1\times10^{-1}$

Sol9. $PCl_5(g) \longrightarrow PCl_3(g) + Cl_2(g)$ (first order reaction)

$K = \dfrac{2.303}{120}\log\left(\dfrac{50}{10}\right)$

$\therefore K = \dfrac{2.303\times\log5}{120\,min} = \dfrac{2.303\times0.6989}{120} = 0.0134\,min^{-1}$

Rate constant at 300K = $1.34\times10^{-2}\,min^{-1}$ [Nearest integer = 1.0]

Sol10. $6OH^- + Cl^- \longrightarrow ClO_3^- + 3H_2O + 6e^-$

Mole of ClO_3^- or $KClO_3$ = $\dfrac{10}{122.6} = 0.08456$ mol

1 mole of ClO_3^- or $KClO_3$ produced by 6 faraday (F).

$\therefore$ 0.08456 mole of ClO_3^- or $KClO_3$ produced by $(6\times0.08156)F$

$\dfrac{I\times t}{F} = 6\times0.8156$

$I = \dfrac{6\times0.08156\times96500}{10\times60\times60}A = 1.311A \approx 1A$

PART – C (MATHEMATICS)

SECTION - A

Sol1. Equation of plane $\perp^r$ to line $\frac{x-3}{2}=\frac{x-1}{1}=\frac{x-2}{1}$ and passes through the point (2, 3,-1) is 2(x – 2) + 1 (y – 3) + 1 (z + 1) = 0

$\Rightarrow$ 2x + y + z – 6 = 0 ....(i)

Hence point (1,2,2) satisfies equation (i)

Sol2. $z=\frac{1}{1-\cos\theta+2i\sin\theta}=\frac{1-\cos\theta-2i\sin\theta}{(1-\cos\theta)^2-(2i\sin\theta)^2}=\frac{2\sin^2\frac{\theta}{2}-2i\sin\theta}{(1-\cos\theta)^2+4\sin^2\theta}$

$\therefore \text{Re}(z)=\frac{2\sin^2\frac{\theta}{2}}{4\sin^2\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}=\frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)}=\frac{1}{5} \Rightarrow \cos^2\frac{\theta}{2}=\frac{1}{2}$

$\Rightarrow \theta=\frac{\pi}{2} \qquad \therefore \qquad \int_0^{\frac{\pi}{2}}\sin x\,dx=1$

Sol3. $\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & k \end{vmatrix}=0 \Rightarrow k=-5$

Sol4. $(1-y)^m(1+y)^n$

The coefficient of y (a_1) = 1.$^nC_1$ + $^mC_1$ (-1) = n – m = 10 .....(i)

coefficient of y^2(a_2) = 1. $^nC_2$ – $^mC_1$. $^nC_1$. + 1.$^mC_2$ = 10

$\frac{n(n-1)}{2}-mn+\frac{m(m-1)}{2}=10 \Rightarrow m^2+n^2-2mn-(n+m)=20$

$\Rightarrow (n-m)^2-(n+m)=20 \Rightarrow n+m=80$

Sol5. $\tan^{-1}\left(\frac{2\times\frac{3}{5}}{1-\frac{9}{25}}\right)+\sin^{-1}\frac{5}{13} \Rightarrow \tan^{-1}\frac{15}{8}+\tan^{-1}\frac{5}{12}=\tan^{-1}\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\frac{5}{12}}=\tan^{-1}\frac{220}{21}$

$\therefore \tan\left(\tan^{-1}\frac{220}{21}\right)=\frac{220}{21}$

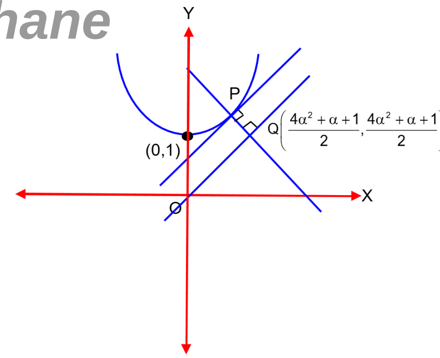
Sol6. $I=\sum_{r=0}^{n-1}f\left(\frac{5r}{n}\right)\frac{1}{n} \Rightarrow I=\int_0^1 f(5x)dx=\int_0^1(5x+1)dx=\frac{7}{2}$

Sol7. Let mid point of PQ is R(h,k)

$$\therefore\ h = \frac{\alpha + \frac{4\alpha^2+\alpha+1}{2}}{2} \text{ and}$$

$$k = \frac{4\alpha^2+1+\frac{4\alpha^2+\alpha+1}{2}}{2}$$

Eliminate α from above these two, we get
$2(3x - \quad y)^2 + (x - 3y) + 2 = 0$



Sol8. $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ ……………………(i)

$f'(x) = 3x^2 - 6x - \frac{3f''(2)}{2}$ ……………..(ii)

$f''(x) = 6x - 6$ ……………..(iii)

$\therefore f''(2) = 12 - 6 = 6$

Use (ii) $f''(x) = 6x - 6$

$f''(2) = 12 - 6 = 6$

$f''(-1) = -6 - 6 < 0$

Hence f(x) is local maxima at x = –1

and f(x) is local minima at x = 3

$\therefore$ from (i) local minimum value at x = 3 is f(3) = –27

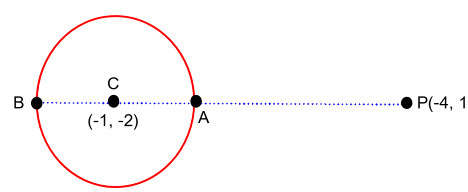
Sol9. Center of the smallest circle is A

Center of the largest circle is B

$r_1 = |CP + CB| = 3\sqrt{2} + 3$ and

$r_2 = |CP - CB| = 3\sqrt{2} - 3$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = 3 + 2\sqrt{2}$$



Sol10. $10 = \frac{7+10+11+15+a+b}{6} \Rightarrow a+b = 17$ ……………………..(i)

$$\Rightarrow \frac{20}{3} = \frac{7^2+10^2+11^2+15^2+a^2+b^2}{6} - 10^2$$

$a^2 + b^2 = 145$ ……………………..(ii)

solving (i) and (ii) we get a = 8, b = 9 or a = 9 , b = 8

$|a - b| = 1$

Sol11. $A = -\frac{y}{x} + 2\sin x + 2$

$\therefore \frac{dy}{dx} + \frac{y}{x} = 2\sin x + 2$……………..(i)

$\Rightarrow I.F = e^{\int \ln x dx} = x$ from (i) $d(xy) = 2\int x \sin x dx + 2\int x dx$

$xy = 2[-x\cos x + \sin x] + x^2 + c$ ………………..(ii) according to question, we get c = 0 $\therefore y\left(\frac{\pi}{2}\right)=\frac{4}{\pi}+\frac{\pi}{2}$

Sol12. $f(x)=\frac{5x+3}{6x-\alpha}=y$ ....(i)

$\Rightarrow x=\frac{\alpha y+3}{6y-5}\Rightarrow f^{-1}(x)=\frac{\alpha x+3}{6x-5}$ ....(ii)

According to question , $f(x)=f^{-1}(x)$ $\therefore$ from (i) and (ii) we get $\propto = 5$

Sol13. $\frac{x+1}{a}=\frac{y}{1}=\frac{z-1}{a}$ ....(i)

$\frac{x+2}{a}=\frac{y}{1}=\frac{z}{\frac{3}{b}}$ ....(ii)

Let A(-1,0,1) and B(-2,0,0) $\therefore$ direction ratios of AB =-1,0,-1

$\because$ lines are coplanar

$\therefore \begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix}=0 \Rightarrow$ b = 1 and a $\in$ R-{0}

Sol14. The projection of $\overrightarrow{BA}$ on $\overrightarrow{BC}=\cos\angle ABC=7\left|\frac{7^2+3^2-5^2}{2\times7\times3}\right|=\frac{11}{2}$

Sol15. $\because f(x)=\ln\left(x+\sqrt{1+x^2}\right)\therefore f(-x)=-f(x)$.

Hence f(x) is an odd function .

Now $g(t)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$

Put $t=0, g(0)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(f(x))dx=2\int_{0}^{\frac{\pi}{2}}\cos(f(x))dx$ ....(i)

where cos(f(x)) is an even function .

Now again put t = 1,

$g(1)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\left(\frac{\pi}{4}+f(x)\right)dx=\frac{1}{\sqrt{2}}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(\cos(f(x))-\sin(f(x))\right)dx$

$= \frac{1}{\sqrt{2}}\left[\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos(f(x))dx - \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin(f(x))dx\right] = \frac{1}{\sqrt{2}} \times 2\int_{0}^{\frac{\pi}{2}} \cos(f(x))dx$, where $\cos(f(x))$ is an even function and $\sin(f(x))$ is an odd function.

$\therefore g(1) = \frac{1}{\sqrt{2}} \times g(0)$, from (i)

$\therefore g(0) = \sqrt{2}g(1)$

Sol16. Truth table
Hence according to option
(4) is most appropriate option

p	q	p →q
T	T	T
T	F	F
F	T	T
F	F	T

Sol17. $\log_{9^{\frac{1}{2}}} x + \log_{9^{\frac{1}{3}}} x + \log_{9^{\frac{1}{4}}} x + ... + \log_{9^{\frac{1}{22}}} x = 504$

$\Rightarrow 2\log_9 x + 3\log_9 x + 4\log_9 x + ... + 22\log_9 x = 504$

$\Rightarrow \log_9 x + 2\log_9 x + 3\log_9 x + 4\log_9 x + ... + 22\log_9 x - \log_9 x = 504$

$\Rightarrow (1+2+3+......+22)\log_9 x - \log_9 x = 504 \Rightarrow x = 81$

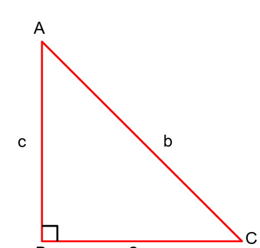
Sol18. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([x] + [-\sin x])dx$ ....(i)

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([-x] + [\sin x])dx$ ....(ii)

by using property $\left(\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

Adding (i) and (ii) we get $2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-2)dx = -2\pi \Rightarrow I = -\pi$

Sol19. Let smallest angle is C = θ
Therefore angle A = $90^0 - \theta$
And angle B = 90^0
i.e. b > a > c
Here , a =2Rcosθ , b = 2R , c = 2Rsinθ
$b^2 = a^2 + c^2$
according to question ,



$\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$

$. \Rightarrow \frac{a^2b^2}{c^2} = a^2 + b^2$

$$\Rightarrow \frac{\cos^2\theta}{\sin^2\theta} = \cos^2\theta + 1$$

$$\Rightarrow 1 - \sin^2\theta = \sin^2\theta\left(2 - \sin^2\theta\right)$$

$$\Rightarrow \sin^4\theta - 3\sin^2\theta + 1 = 0$$

$$\Rightarrow \sin^2\theta = \frac{3-\sqrt{5}}{2}$$

$$\Rightarrow \sin\theta = -\frac{\sqrt{5}-1}{2}$$

Sol20. $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 1 - k$,

$P(\bar{A} \cap C) + P(A \cap \bar{C}) = 1 - 2k$,

$P(\bar{B} \cap C) + P(B \cap \bar{C}) = 1 - k$ and

$P(A \cap B \cap C) = k^2$

$P(A) + P(B) - 2P(A \cap B) = 1 - k$ ....(i)

$P(B) + P(C) - 2P(B \cap C) = 1 - k$ ....(ii)

$P(A) + P(C) - 2P(A \cap C) = 1 - 2k$ ....(iii)

Adding (i),(ii) and (iii) we get $P(A \cup B \cup C) = \frac{-4k+3}{2} + k^2$

$$\Rightarrow P(A \cup B \cup C) = \frac{2k^2 - 4k + 2 + 1}{2} = \frac{2(k-1)^2 + 1}{2} \Rightarrow P(A \cup B \cup C) > \frac{1}{2}$$

SECTION - B

Sol1. $a_{n+1} = \frac{a_{n+2} - a_n}{2}$ let $p = \sum_{n=1}^{\infty} \frac{a_n}{8^n}$

$$\Rightarrow 64\frac{a_{n+2}}{8^{n+2}} = \frac{16a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$$

$$\Rightarrow 64\sum_{n=1}^{\infty}\frac{a_{n+2}}{8^{n+2}} = \sum_{n=1}^{\infty}\frac{16a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty}\frac{a_n}{8^n} \Rightarrow 64\left(p - \frac{a_1}{8} - \frac{a_2}{8^2}\right) = 16\left(p - \frac{a_1}{8}\right) + p$$

$$\Rightarrow 64\left(p - \frac{1}{8} - \frac{1}{8^2}\right) = 16\left(p - \frac{1}{8}\right) + p \Rightarrow 64p - 8 - 1 = 16p - 2 + p \qquad \Rightarrow 47p = 7$$

Sol2. $$\lim_{x\to 0}\frac{\alpha x\left(1 + x + \frac{x^2}{2} + ...\right) - \beta\left(x - \frac{x^2}{2} + \frac{x^3}{3} + ...\right) + \gamma x^2(1-x)}{x^3} = 10$$

For limit to exist $\alpha - \beta = 0$ ....(i) , $\alpha + \frac{\beta}{2} + \gamma = 0$ ....(ii)

and $\frac{\alpha}{2} - \frac{\beta}{2} - \gamma = 10$ ....(iii)

Solving (i),(ii) and (iii) we get $\alpha = 6, \beta = 6$ and $\lambda = -9 \Rightarrow \alpha + \beta + \lambda = 3$

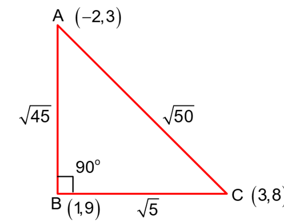
Sol3. $\left(\sqrt{50}\right)^2=\left(\sqrt{45}\right)^2+\left(\sqrt{5}\right)^2$

$\therefore \angle B=90^\circ$ circum centre = $O\left(\frac{1}{2},\frac{11}{2}\right)$

Mid point of $BC=D\left(2,\frac{17}{2}\right)$

Equation of OD is $y=2x+\frac{9}{2}$. This line passes through

$\left(0,\frac{\alpha}{2}\right)\therefore \alpha=9$



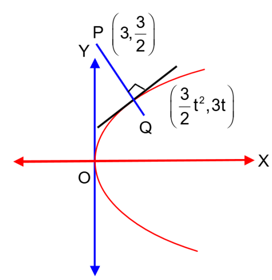
Sol4. Let equation of normal PQ is

$y=-tx+3t+\frac{3}{2}t^3$ and it is passing through

$P\left(3,\frac{3}{2}\right)\therefore t=1$

$\therefore Q\left(\frac{3}{2},3\right)\Rightarrow a=\frac{3}{2}$ and $b=3$

$2(a+b)=9$



Sol5. $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}\Rightarrow |A|=4\Rightarrow \left|3\operatorname{adj}\left(2A^{-1}\right)\right|=\left|3.2^2\operatorname{adj}\left(A^{-1}\right)\right|$

$\Rightarrow 12^3\left|\operatorname{adj}\left(A^{-1}\right)\right|=12^3\left|A^{-1}\right|^2=\frac{12^3}{|A|^2}=\frac{12\times12\times12}{16}=108$

Sol6. $\because\ |\vec{v}_1|=|\vec{v}_2|\Rightarrow p^2-p-2=0\Rightarrow p=-1,2$ we take only $p=2$ $(p>0)$

$\therefore\ \cos\theta=\frac{\vec{v}_2.\vec{v}_2}{|\vec{v}_1||\vec{v}_2|}=\frac{4\sqrt{3}+3}{13}\Rightarrow \tan\theta=\frac{6\sqrt{3}-2}{4\sqrt{3}+3}=\frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}\therefore \alpha=6$

Sol7. $\int dy=\int\frac{\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)}{e^x\sqrt{1-(e^{-x})^2}}dx$ put $\cos2\theta=e^{-x}\Rightarrow -2\sin2\theta d\theta=-e^{-x}dx$

$\therefore \int dy=\int\frac{2\cos\theta\sin2\theta d\theta}{\sqrt{1-\cos^2 2\theta}}\Rightarrow y=2\sin\theta+c\Rightarrow y=-1,\ \theta=0,\ c=0$

$\therefore\ y=2\sqrt{\frac{1-e^{-x}}{2}}-1$ at $(\alpha,0), e^{\alpha}=2$

Sol8. $\frac{\log((2x+5)(x+1))}{\log(x+1)}+\frac{\log(x+1)}{\log(2x+5)}-4=0\Rightarrow \frac{\log(2x+5)}{\log(x+1)}+\frac{\log(x+1)}{\log(2x+5)}-3=0$

$\Rightarrow x\in(-1,0)\cup(0,\infty)$

Put $t=\dfrac{\log(2x+5)}{\log(x+1)} \Rightarrow t+\dfrac{2}{t}-3=0 \Rightarrow t^2-3t+2=0 \Rightarrow t=1,2$

$\dfrac{\log(2x+5)}{\log(x+1)}=1 \Rightarrow x=-4$ (not possible) and $\dfrac{\log(2x+5)}{\log(x+1)}=2 \Rightarrow x=2$

Hence only one solution is possible.

Sol9. $\dfrac{1}{\alpha(\alpha+1)(\alpha+2)..........(\alpha+20)}=\dfrac{A_0}{\alpha}+\dfrac{A_1}{\alpha+1}+\dfrac{A_2}{\alpha+2}+........+\dfrac{A_{20}}{\alpha+20}$

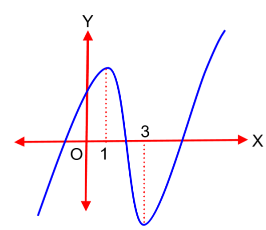
Solving by partial fraction , we get

$A_{13}=\dfrac{-1}{14!7!}, A_{14}=\dfrac{-1}{14!6!}$ and $A_{15}=\dfrac{-1}{14!5!}$

$\therefore A_{14}+A_{15}=\dfrac{1}{14!6!}-\dfrac{1}{15!5!}=\dfrac{1}{14!5!}\times\dfrac{1}{10}$

$\therefore \dfrac{A_{14}+A_{15}}{A_{13}}=\dfrac{3}{10} \Rightarrow \left(\dfrac{A_{14}+A_{15}}{A_{13}}\right)^2=\left(\dfrac{3}{10}\right)^2 \Rightarrow 100\left(\dfrac{A_{14}+A_{15}}{A_{13}}\right)^2=9$

Sol10. $f(t)=t^3-6t^2+9t+3$



$\Rightarrow f'(t)=3t^2-12t+9=3(t-1)(t-3)$

$\therefore f'(t)=0 \Rightarrow t=1,3$

$\Rightarrow f(1)=1, f(3)=-3$

$g(x)=\begin{cases} f(x) & , \quad 0\le x<1 \\ 1 & , \quad 1\le x\le 3 \\ 4-x & , \quad 3<x\le 4 \end{cases}$ g(x) is continuous

$g'(x)=\begin{cases} 3(x-1)(x-3) & , \quad 0\le x<1 \\ 0 & , \quad 1\le x\le 3 \\ -1 & , \quad 3<x\le 4 \end{cases}$.

Hence g(x) is not differentiable at only x = 3.